Course 16
Geometric Data Structures for Computer Graphics

Quadtrees

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Set of points, initial square $Q$ and root $R$ of the tree.

Subdivision into quadrants in counterclockwise order.

Recursively, until square has $\leq 1$ objects.

Node $v$ represents square $Q(v)$.

Recursive construction of the tree: given points/initial square.
Definition Quadtree

- Rooted tree
- Internal nodes have 4 children
- Every node represents a square
- Children represent subsquares of the square
- Geometric data of squares (leaves): Points, Lines, Rectangles, Ellipses
- Octree → 8 children, Boxes, higher Dimensions
Properties

- Quadtrees of depth $d$ with $n$ points
- Number of nodes: $\mathcal{O}(dn)$
  - Number of leaves: $3 \times \# \text{Internal nodes} + 1$
  - At every depth only $n$ internal nodes
- Construction: $\mathcal{O}(dn)$ time
  - Every depth in the recursive construction
  - Distribution of points: Linear in the number of points
- Depth of the quadtree depends on distances of objects:
  - Let $c$ be the distance of the closest pair
  - Let $s$ be the side length of the initial $Q$
  - Depth $d \leq \log(s/c) + \frac{3}{2}$
- Balancing depend on objects
Application Nearest Neighbors

• Compute List of Nearest Neighbors of a query point $q$
• Idea: Observe Neighboring quadrants recursively
  – Find quadrant of query point $q$ in $O(d)$ time
  – Build Priority-Queue $P$: Visited squares sorted by distance in $O(d \log d)$ time
  – Iteration
    * Take first (closest) square/object $O$ of $P$
    * Object: $\Rightarrow$ report
    * Square: $\Rightarrow$ insert subsquares or single object into $P$
    * Delete $O$ out of $P$
    * Repeat until $P$ is empty
• Time: $O(n \log n)$

Nearest Neighbor Applet by F. Brabec and H. Samet